PROPAGATION OF SOUND IN SUSPENSIONS

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The results of papers [1-3] are generalized to include the case of ultrasonic frequencies.

Historically, the theory of sound diffraction by a sphere has been developed for the purpose of determining its absorption. Dispersion, in the past, has attracted considerably less attention. Papers [1-3], published recently, show the increasing interest toward this subject.

The relaxation theory used in [1-3] requires a precise calculation of the reactions of particles on the oscillations of the surrounding medium. The force acting on a particle in a sonic field has been found already by Stokes [4]. Our interest will be focussed on the investigation of temperature effects.

First of all, from calculation of the diffraction of a plane sound wave at a sphere [5, 6] by the procedure in [7], the thermal dispersion of sound is investigated in emulsions with constituents which have similar mechanical properties. The results obtained, taking account of thermal conductivity inside each constituent, are systematized on the basis of the function F, well known in the literature [5]. The average laws of conservation of the mixture are then written on the assumption that the effects of viscosity and thermal conductivity are important only in phase interaction processes, and on this basis the acoustic properties of various hydro- and gas-mixtures are studied.

Comparison of the results from both cases allows the general frequency relation to be established for the coefficient of damping and the velocity of sound for suspensions of very different nature and also estimates the effect which is introduced by thermal conductivity inside each of the constituents individually.

1. We consider a plane compression wave incident on a spherical surface of radius R. We shall assume that the particle size is much less than the wavelength of the primary wave. We shall use the spherical coordinates r, ϑ, and ψ to represent the field, taking the origin at the center of the sphere and the polar axis in the direction of propagation of the incident wave. We express the potential of the latter $\varphi_0 = \exp(ikr\cos\vartheta)$ in the usual way*

$$\varphi_0 = \sum_{n=0}^{\infty} i^n (2n+1) j_n(kr) P_n(\cos\vartheta).$$

Because of the presence of the sphere, six secondary waves originate: three in the primary medium – compression wave φ_r, temperature wave Φ, and viscous wave **A** – and three inside the particle. The velocity resulting from the field scattered by the droplet can always be represented by the expression

$$\mathbf{u} = \operatorname{grad}(\varphi_r + \Phi) + \operatorname{rot}\mathbf{A}.$$

In view of the axial symmetry, of the projections A_r, A_ϑ, and A_ψ the first two are equal to zero and it only remains to consider $A_\psi = A$.

The potentials φ_r, Φ, and A will be found in the form

$$\varphi_r = \sum_{n=0}^{\infty} i^n (2n+1) A_n h_n(kr) P_n(\cos\vartheta),$$

*Here, and in future, we shall omit the time-dependent factor $\exp(-i\omega t)$.

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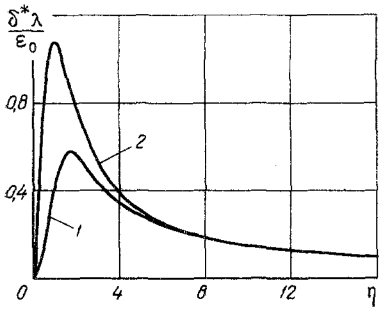

Fig. 1

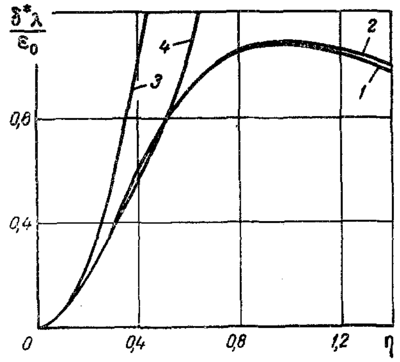

Fig. 2

Fig. 1. Relative absorption of sound: 1) emulsion of benzene in water; 2) emulsion of water in benzene.

Fig. 2. Comparison of values of $\delta^*\lambda/\varepsilon_0$ (emulsion of water in benzene), calculated using the equations: 1) (5); 2) (9); 3) (7); and 4) (8).

$$\Phi = \sum_{n=0}^{\infty} i^n (2n+1) B_n h_n (Kr) P_n (\cos\vartheta),$$
$$A' = \sum_{n=0}^{\infty} i^n (2n+1) C_n h_n (\varkappa r) P_n^1 (\cos\vartheta).$$

Similarly, for the interior of the sphere

$$\varphi' = \sum_{n=0}^{\infty} i^n (2n+1) A_n' j_n (k'r) P_n (\cos\vartheta),$$

$$\Phi' = \sum_{n=0}^{\infty} i^n (2n+1) B_n' j_n (K'r) P_n (\cos\vartheta),$$

$$A' = \sum_{n=0}^{\infty} i^n (2n+1) C_n' j_n (\varkappa' r) P_n^1 (\cos\vartheta).$$

Here, j_n, h_n, P_n, and P_n^1 are spherical functions of Bessel and Hankel, and Legendre functions and adjoint Legendre functions.

The boundary conditions at the surface of the droplet have the form

$$u_r = u_r', \quad u_\vartheta = u_\vartheta', \quad T = T',$$
$$P_{rr} = P_{rr}', \quad P_{r\vartheta} = P_{r\vartheta}', \quad \lambda \frac{\partial T}{\partial r} = \lambda' \frac{\partial T'}{\partial r}.$$

Substituting here the general expressions for φ_r, Φ, . . . , A', we find the unknown coefficients A_n, . . . , C_n'. As only the potentials of the sonic field are unknown, the various physical values can be calculated.

We note that in the boundary conditions for the number n = 0, there is no potential of the viscous wave. This circumstance permits the use of light suspensions by the procedure in [7] for investigating dispersion of the velocity of sound.

We introduce the complex wave number

$$\bar{k} + i\delta^* = \omega \sqrt{\frac{\overline{\rho_0 s}}{\bar{p}}}$$

into the calculation and we find the compression s at every point of space from the equation of state for the component

$$s = \beta p - \alpha T, \quad s' = \beta' p - \alpha' T'.$$

We determine the average compression $\bar{s}$ by the formula:

$$\bar{s} = \frac{1}{V+V'}\left[\int_V s dV + \int_{V'} s' dV'\right].$$

We know the spatial temperature distribution for each of the components from the calculation of the diffraction of waves at a sphere.

A progressive calculation gives for the wave number

$$\bar{k} + i\delta^* = \bar{k}_\infty + i\frac{3}{2R^2}\varepsilon_0\lambda T_0\bar{\rho}_0\bar{q}_\infty\left(\frac{\alpha}{\rho_0 c_p} - \frac{\alpha'}{\rho_0' c_p'}\right)F^{-1}. \quad (1)$$

Hence

$$\bar{q} = \bar{q}_\infty\left[1 + \frac{3}{4}\varepsilon_0(\gamma-1)\frac{\bar{\rho}_0}{\rho_0}\frac{\bar{q}_\infty^2}{q^2}(1-\zeta)^2\frac{\mathrm{Im}(F^{-1})}{\eta^2}\right], \quad (2)$$

$$\delta^* = \frac{3}{4}\varepsilon_0\omega(\gamma-1)\frac{\bar{\rho}_0}{\rho_0}\frac{\bar{q}_\infty}{q^2}(1-\zeta)^2\frac{\mathrm{Re}(F^{-1})}{\eta^2}, \quad (3)$$

where

$$\bar{q}_\infty = \left\{\bar{\rho}_0\left[\varepsilon_0\frac{\beta'}{\gamma'} + (1-\varepsilon_0)\frac{\beta}{\gamma}\right]\right\}^{-\frac{1}{2}}, \quad (4)$$

$$F = \frac{h_0(KR)}{KRh_1(KR)} - \chi\frac{j_0(K'R)}{K'Rj_1(K'R)}. \quad (5)$$

Conversion from spherical functions in expression (5) to hyperbolic functions gives, for $\eta \gg 1$,

$$F^{-1} = (1-i)\frac{\eta}{1+\sqrt{\chi\rho_0 c_p/\rho_0' c_p'}}. \quad (6)$$

For low frequencies ($\eta \ll 1$), we have from Eq. (5)

$$F^{-1} = \frac{4\eta^2}{9(\rho_0 c_p/\rho_0' c_p')^2}\left[(1+0.2\chi)\eta^2 - i\frac{3}{2}\frac{\rho_0 c_p}{\rho_0' c_p'}\right]. \quad (7)$$

Relations (1) to (7) are identical with the corresponding computations of M. A. Isakovich [7].

It is difficult to evaluate the real and imaginary parts of the general formula (5) and therefore the natural tendency of the author of [6] was to a presentation F, which although simple should include at the same time the maximum possible range of frequencies.

If expression (5) is rewritten in a power sequence, as in [6], then we obtain*

$$F = 1 - \eta + 0.2\chi + i\left[\eta + 2\eta^2 + \chi\left(\frac{1.5}{\eta'^2} + \frac{2}{175}\eta'^2\right)\right]. \quad (8)$$

The limits of applicability of Eq. (8) are somewhat wider than for Eq. (7).

A significant displacement gives the following expression

$$\mathrm{Re}(F^{-1}) = \frac{\{4\eta^4[1+0.2\chi+(1+0.4\chi)\eta] + 1.6\chi\eta^6\}}{\left\{4(1+0.4\chi)\eta^4 + 12\frac{\rho_0 c_p}{\rho_0' c_p'}\eta^3 + 9\left(\frac{\rho_0 c_p}{\rho_0' c_p'}\right)^2(1+2\eta+2\eta^2) + 1.6\chi\eta^5\right\}}, \quad (9)$$

$$\mathrm{Im}(F^{-1}) = -\frac{\left\{\eta^2\left[4\eta^3 + 6\frac{\rho_0 c_p}{\rho_0' c_p'}(1+2\eta+2\eta^2)\right]\right\}}{\left\{4(1+0.4\chi)\eta^4 + 12\frac{\rho_0 c_p}{\rho_0' c_p'}\eta^3 + 9\left(\frac{\rho_0 c_p}{\rho_0' c_p'}\right)^2(1+2\eta+2\eta^2) + 1.6\chi\eta^5\right\}}. \quad (10)$$

The results of the calculations are given in Figs. 1 and 2.

*The similar formula in [6] is written incorrectly.

2. In the case of heavy suspensions, it is necessary to take account of viscosity. The system of equations describing the acoustic process in a mixture with rigid particles has the form [3]:

$$\frac{\partial}{\partial t}[(1-\varepsilon_0)\rho-\rho_0\varepsilon]+(1-\varepsilon_0)\rho_0\frac{\partial u}{\partial x}=0,$$
$$\frac{\partial\varepsilon}{\partial t}+\varepsilon_0\frac{\partial u'}{\partial x}=0,$$
$$\frac{\partial}{\partial t}[(1-\varepsilon_0)\rho_0 u+\varepsilon_0\rho_0' u']=-\frac{\partial p}{\partial x},$$
$$V\frac{\partial}{\partial t}(\rho_0' u'-\rho_0 u)=F_p, \tag{11}$$
$$\frac{\partial}{\partial t}[(1-\varepsilon_0)\rho_0 c_p T+\varepsilon_0\rho_0' c_p' T'-(1-\varepsilon_0)\alpha T_0 p]=0,$$
$$V\frac{\partial}{\partial t}(\rho_0' c_p' T'-\rho_0 c_p T)=Q_p,$$
$$\frac{\rho}{\rho_0}=\beta p-\alpha T.$$

Here F_p is the reaction encountered by the particles during motion relative to the medium. The latter was calculated by Stokes [4] and, for harmonic oscillations of the medium $u\sim\exp(-i\omega t)$, has the form

$$F_p=\left(-\frac{V\rho_0}{2}i\omega+6\pi R\mu-6\pi R^2\sqrt{-i\omega\rho_0\mu}\right)(u-u'). \tag{12}$$

For a known acoustic field (Section 1), it is not difficult to estimate the quantity Q_p:

$$Q_p=\left(-\frac{V\rho_0 c_p}{2}i\omega+4\pi R\lambda-4\pi R^2\sqrt{-i\omega\rho_0 c_p\lambda}\right)(T-T'). \tag{13}$$

The second and third terms of formula (13) represent the amount of heat transferred per unit time from the medium to the particle; they are the results of evaluating the expression $4\pi R^2\lambda(\partial T/\partial r)|_{r=R}$.

The coefficient $V\rho_0 c_p/2$ for the derivative $(\partial/\partial t)(T-T')$ in the first term is not the same as the specific heat of the "combined" mass.

Assuming that the independent variables in system (11) are proportional to $\exp[i(kx-\omega t)]$, we obtain†

$$\frac{\bar{k}^2}{\omega^2}=\frac{(1-\varepsilon_0)\beta\rho_{ef}}{\gamma_{ef}}, \tag{14}$$

where

$$\rho_{ef}=\rho_0\frac{1+\frac{\varepsilon_0}{1-\varepsilon_0}\delta B_\mu}{1+\frac{\varepsilon_0}{1-\varepsilon_0}B_\mu}; \tag{15}$$

$$\gamma_{ef}=\frac{c_p+\frac{\varepsilon_0}{1-\varepsilon_0}\delta c_p' B_\lambda}{c_V+\frac{\varepsilon_0}{1-\varepsilon_0}\delta c_p' B_\lambda}; \tag{16}$$

$$B_\mu=\frac{u'}{u}=\frac{1+\xi-i\xi\left(1+\frac{2}{3}\xi\right)}{1+\xi-i\xi\left[1+\frac{2}{9}(1+2\delta)\xi\right]}, \tag{17}$$

$$B_\lambda=\frac{T'}{T}=\frac{1+\eta-i\eta(1+\eta)}{1+\eta-i\eta\left[1+\frac{1}{3}\left(1+2\delta\frac{c_p'}{c_p}\right)\eta\right]}. \tag{18}$$

†Having used the case, we note that in [3] all computations are carried out for the relation $f=f^*\exp[i(\omega t-kx)]$, and in symbols $\tau_\mu=(2/9)(\rho_M r^2/\mu)$.

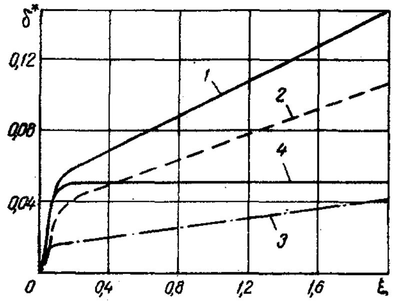

Fig. 3

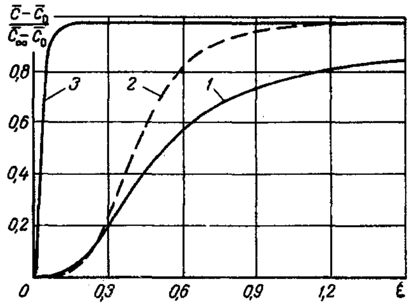

Fig. 4

Fig. 3. Absorption of sound δ^*, dB/m, by water droplets in air: 1) total, formulas (20), (21), and (22); 2) viscous, formula (21); 3) thermal, formula (22); 4) total, formulas (20), (23), and (24).

Fig. 4. Calculation of the dispersion of the velocity of sound. 1, 2) Emulsion of mercury in water, formulas (19), (21), (22), and (19), (23), and (24); 3) suspension of aluminum particles in air.

The asymptotic estimates for the velocity of sound follow from Eqs. (14)-(18)

$$\bar{q}_0 = \left[\frac{(1-\varepsilon_0)\beta\bar{\rho}_0}{\bar{\gamma}_0}\right]^{-\frac{1}{2}},$$

$$\bar{q}_\infty = \left[\frac{(1-\varepsilon_0)\beta\bar{\rho}_\infty}{\gamma}\right]^{-\frac{1}{2}},$$

where

$$\bar{\gamma}_0 = \frac{c_p + \dfrac{\varepsilon_0}{1-\varepsilon_0}\delta c_p'}{c_V + \dfrac{\varepsilon_0}{1-\varepsilon_0}\delta c_p'},$$

and for $\bar{\rho}_\infty$ under the conditions of [1, 3] we have

$$\frac{1}{\bar{\rho}_\infty} = \frac{\varepsilon_0}{\rho_0'} + \frac{1-\varepsilon_0}{\rho_0}$$

("parallel coupling" for densities). A more rigorous estimate for $\bar{\rho}_\infty$ can be obtained from Eqs. (15) and (17). We consider rarefied systems in which the density $\bar{\rho}_\infty$ is almost equal to the density of the containing medium ρ_0.

Assuming that dispersion is small, we find from Eqs. (14)-(18)

$$\frac{1}{\bar{q}} = \frac{1}{\bar{q}_0}\left[1 + \frac{1}{2}\frac{\varepsilon_0(\delta-1)}{1+\varepsilon_0(\delta-1)}\operatorname{Re}(B_\mu - 1) + \frac{1}{2}\frac{\varepsilon_0(\gamma-1)\delta c_p' c_V}{(c_p+\varepsilon_0\delta c_p')(c_V+\varepsilon_0\delta c_p')}\operatorname{Re}(B_\lambda - 1)\right], \tag{19}$$

$$\delta^* = \frac{\omega}{\bar{q}_0}\left[\frac{1}{2}\frac{\varepsilon_0(\delta-1)}{1+\varepsilon_0(\delta-1)}\operatorname{Im}(B_\mu - 1) + \frac{1}{2}\frac{\varepsilon_0(\gamma-1)\delta c_p' c_V}{(c_p+\varepsilon_0\delta c_p')(c_V+\varepsilon_0\delta c_p')}\operatorname{Im}(B_\lambda - 1)\right]. \tag{20}$$

Here,

$$B_\mu - 1 = \frac{-\dfrac{4}{9}(\delta-1)\xi^3\left[1+\dfrac{2}{9}(1+2\delta)\xi\right] + i\dfrac{4}{9}(\delta-1)\xi^2(1+\xi)}{(1+\xi)^2+\xi^2\left[1+\dfrac{2}{9}(1+2\delta)\xi\right]^2}; \tag{21}$$

$$B_\lambda - 1 = \frac{-\dfrac{2}{3}\delta\dfrac{c_p'}{c_p}\eta^3\left[1+\dfrac{1}{3}\left(1+2\delta\dfrac{c_p'}{c_p}\right)\eta\right] + i\dfrac{2}{3}\delta\dfrac{c_p'}{c_p}\eta^2(1+\eta)}{(1+\eta)^2+\eta^2\left[1+\dfrac{1}{3}\left(1+2\delta\dfrac{c_p'}{c_p}\right)\eta\right]^2}. \tag{22}$$

For small values of ξ, η and $\delta \gg 1$, Eqs. (21) and (22) convert to the corresponding formulas of paper [3]:

$$B_\mu - 1 = \frac{-\left(\frac{4}{9}\right)^2 \delta(\delta-1)\xi^4 + i\frac{4}{9}(\delta-1)\xi^2}{1+\left(\frac{4}{9}\delta\right)^2\xi^4}, \tag{23}$$

$$B_\lambda - 1 = \frac{-\left(\frac{2}{3}\delta\frac{c_p'}{c_p}\right)^2\eta^4 + i\frac{2}{3}\delta\frac{c_p'}{c_p}\eta^2}{1+\left(\frac{2}{3}\delta\frac{c_p'}{c_p}\right)^2\eta^4}. \tag{24}$$

The change δ^* in the function $\xi(\eta = \mathrm{Pr}^{1/2}\xi)$ for a suspension of water droplets in air is shown in Fig. 3. It is assumed for the calculation that $R = 5 \cdot 10^{-6}$ m, $q = 344$ m/sec and $\varepsilon_0 = 10^{-6}$. δ^* increases monotonically with increase of ξ at first proportional to ξ^4 and then, after inflexion, to ξ. This latter circumstance follows on taking account of the inertial terms (terms which are frequency-dependent) in formulas (12) and (13). Without taking account of the latter (see [1-3]), at high frequencies we should have $\delta^* = \mathrm{const}$ (curve 4, Fig. 3).

Analysis of the dispersion of sound shows that in the case of a gas suspension ($\delta \gg 1$, $\delta(c_p'/c_p) \gg 1$) the inertial terms do not carry any significant correction to the velocity of sound. The dispersion curves, calculated using formulas (21) and (22) and the simplified formulas (23) and (24), are plotted one on the other (curve 3, Fig. 4).

We observe a different pattern when considering a hydrosuspension. For example, let us take an emulsion of mercury in water. Starting at approximately $\xi = 0.3$, curves 1 and 2 (Fig. 4) diverge so strongly that there can be no question of using formulas (23) and (24).

In conclusion we note that in the formulation of system (11), we have neglected thermal conductivity inside each of the phases individually. This effect of conductivity has a clearly defined form and is represented in Eq. (9) and (10) by the terms containing χ. When $\chi \to 0$, it vanishes and

$$F^{-1} \to -i\frac{2}{3}\delta\frac{c_p'}{c_p}\eta^2 B_\lambda.$$

The results of the theory discussed are used over a whole range of frequencies, including high ultrasonic frequencies and they are found to coincide with the experimental and calculated data of [2, 5, 8, 11-15].

NOTATION

σ	is the coefficient of thermal conductivity (thermal diffusivity);
q	is the velocity of sound;
c_p, c_v	are the specific heats at constant pressure and constant volume;
$k = \omega/q$, $K = (1+i)(\omega/2\sigma)^{1/2}$, and $\varkappa = (1+i)(\omega/2\nu)^{1/2}$	are the wave numbers of the acoustic, thermal, and viscous waves;
p	is the acoustic pressure;
P_{ij}	is the stress tensor;
R	is the radius of particle;
t	is the time;
T	is the temperature;
u	is the velocity;
V	is the volume of particle; volume occupied by a component;
α	is the coefficient of volume expansion;
β	is the isothermal coefficient of compressibility;
$\gamma = c_p/c_v$	is the ratio of specific heats;
$\delta = \rho_0'/\rho_0$	is the ratio of densities;
δ^*	is the coefficient of absorption;
ε	is the volume concentration;
$\zeta = (\alpha'/\alpha)(\rho_0 c_p/\rho_0' c_p')$,	
$\eta = (\omega R^2/2\sigma)^{1/2}$,	
$\xi = (\omega R^2/2\nu)^{1/2}$,	

$\chi = \lambda/\lambda'$	are parameters;
λ	is the coefficient of thermal conductivity and wavelength;
μ, ν	are the dynamic and kinematic viscosities;
ρ	is the density;
ω	is the angular frequency;
Pr	is the Prandtl number;

Subscripts

0	is the unperturbed value of a quantity; value of a quantity at low frequencies;
∞	is the value of a quantity at high frequencies;
$-$	is the average with respect to volume of mixtures;
'	is the value of a quantity for the suspended phase.

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